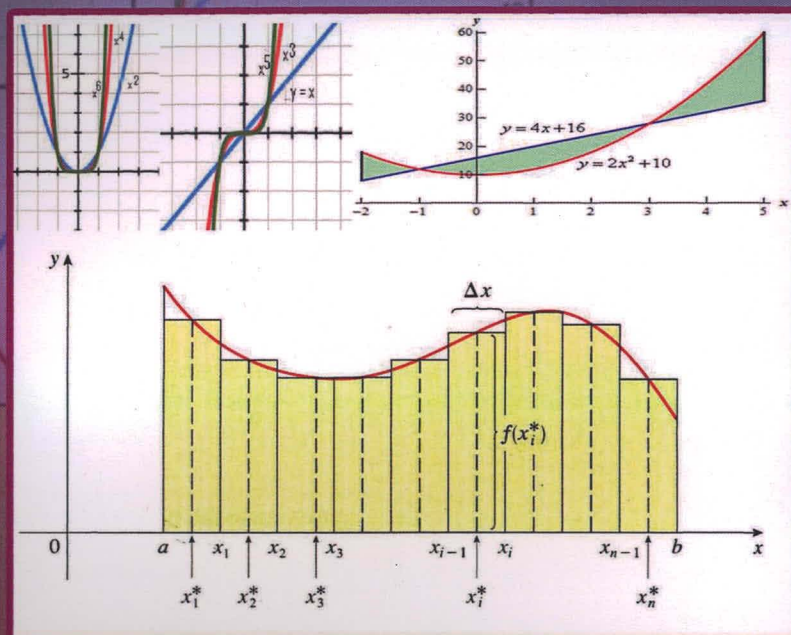


CALCULUS WITH SINGLE VARIABLE



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CALCULUS WITH SINGLE VARIABLE

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Table of Content

| | | |
|---|--|--------------|
| PREFACE | | i |
| ACKNOWLEDGEMENTS | | ii |
| <hr/> | | |
| CHAPTER 0: PRELIMINARIES | By M. Azram and Messikh Azeddin | 1-42 |
| 0.1 | Number System and Inequalities | 1 |
| 0.2 | Functions | 8 |
| 0.3 | Trigonometric and Inverse Trigonometric Functions | 14 |
| 0.4 | Exponential and Logarithmic Function | 28 |
| 0.5 | Linear Transformations | 31 |
| | Exercise 0 | 38 |
| <hr/> | | |
| CHAPTER 1: LIMITS AND CONTINUITY | By Gharib Subhi Mahmoud Ahmad and Raihan Othman | 43-66 |
| 1.1 | Introduction. | 43 |
| 1.2 | Computation of Limits | 43 |
| 1.3 | Calculating Limits Using the Limit Laws | 48 |
| 1.4 | Infinite Limits and Limits at Infinity | 50 |
| 1.5 | Tangent Limits and Derivatives | 54 |
| 1.6 | Continuity | 60 |
| | Exercise 1 | 64 |
| <hr/> | | |
| CHAPTER 2: THE DERIVATIVES | By Jamal I. Daoud | 67-91 |
| 2.1 | The Slope of a Straight Line | 67 |
| 2.2 | Properties of the Slope of a Line | 68 |
| 2.3 | The Slope as a Rate of Change | 70 |
| 2.4 | The Slope of a Curve at a Point | 71 |
| 2.5 | The Derivative using the Definition | 72 |
| 2.6 | Differentiation Rules | 75 |
| 2.6.1 | Derivative Constant Multiple Rule | 76 |
| 2.6.2 | Derivative Sum and Difference Rule | 76 |
| 2.6.3 | Derivative Product Rule | 78 |
| 2.6.4 | Derivative Quotient Rule | 79 |
| 2.7 | Second-And Higher-Order Derivatives | 80 |

| | | |
|-------|---------------------------------------|----|
| 2.8 | Derivative of Trigonometric Functions | 81 |
| 2.8.1 | Derivative of the Sine Function | 81 |
| 2.8.2 | Derivative of Cosine Function | 82 |
| 2.9 | The Chain Rule | 85 |
| 2.10 | Implicit differentiation | 87 |
| | Exercise 2 | 89 |

| | | | |
|-------------------|--|-------------------------|---------------|
| CHAPTER 3: | APPLICATIONS OF DIFFERENTIATION | By Zaharah Wahid | 92-136 |
|-------------------|--|-------------------------|---------------|

| | | |
|-------|---|-----|
| 3.1 | Linear Approximations and Newton's Method | 92 |
| 3.1.1 | Small Changes and Linear Approximations | 92 |
| 3.1.2 | Rates of Change | 97 |
| 3.1.3 | Newton's Method | 99 |
| 3.2 | Indeterminate Forms | 102 |
| 3.2.1 | Function Forms $0/0$, ∞/∞ | 102 |
| 3.2.2 | L' Hospital's Rule or Theorem | 102 |
| 3.3 | Maximum and Minimum Values | 105 |
| 3.3.1 | Increasing and Decreasing Functions | 105 |
| 3.3.2 | Turning Points of a Curve | 108 |
| 3.3.3 | Maximum and Minimum Problem | 112 |
| 3.4 | First and the Second Derivative Tests and Curve Sketching | 115 |
| 3.4.1 | First derivative Test Increasing and Decreasing | 116 |
| 3.4.2 | Concavity and The second derivative Test | 119 |
| 3.4.3 | Second Derivative Test for Local Extrema | 120 |
| 3.4.4 | Distance, Velocity, and Acceleration | 122 |
| 3.4.5 | Instantaneous Velocity | 123 |
| 3.4.6 | Acceleration | 124 |
| 3.5 | Optimization Problems | 127 |
| 3.5.1 | Guideline for solving optimization | 128 |
| 3.5.2 | Applications | 128 |
| | Exercise 3 | 133 |

| | | | |
|-------------------|--------------------|------------------------------|----------------|
| CHAPTER 4: | INTEGRATION | By M. S. H. Chowdhury | 137-168 |
|-------------------|--------------------|------------------------------|----------------|

| | | |
|-----|-----------------------------------|-----|
| 4.1 | Indefinite Integral | 137 |
| 4.2 | Sums and Sigma Notation | 142 |
| 4.3 | Area | 146 |
| 4.4 | Reimann Sum and Definite Integral | 150 |

| | | |
|-----|-------------------------------------|-----|
| 4.5 | The Fundamental Theorem of Calculus | 157 |
| 4.6 | Integration by Substitution | 162 |
| | Exercise 4 | 166 |

| | | | |
|-------------------|---|-----------------------|----------------|
| CHAPTER 5: | APPLICATIONS OF DEFINITE INTEGRALS | By Sellami Ali | 169-200 |
|-------------------|---|-----------------------|----------------|

| | | |
|-----|-----------------------------------|-----|
| 5.1 | Areas Under Curve | 169 |
| 5.2 | Area between Curves | 171 |
| 5.3 | Volume: Slicing Disks and Washers | 176 |
| 5.4 | Arc Length and Surface Area | 189 |
| 5.5 | Work | 193 |
| | Exercise 5 | 197 |

| | | | |
|-------------------|-------------------------------|------------------------------|----------------|
| CHAPTER 6: | INTEGRATION TECHNIQUES | By M. S. H. Chowdhury | 201-245 |
|-------------------|-------------------------------|------------------------------|----------------|

| | | |
|-----|---|-----|
| 6.1 | Review of Integration Using Basic Formulas and Substitution | 201 |
| 6.2 | Integration by Parts | 207 |
| 6.3 | Trigonometric Techniques of Integration | 214 |
| 6.4 | Special Types of Integration | 228 |
| 6.5 | Integration of Rational Functions Using Partial Fractions | 232 |
| 6.5 | Improper Integrals | 236 |
| | Exercise 6 | 243 |

| | | | |
|-------------------|------------------------|---------------------------------------|----------------|
| CHAPTER 7: | INFINITE SERIES | By Faiz A. Elfaki and M. Azram | 246-266 |
|-------------------|------------------------|---------------------------------------|----------------|

| | | |
|-------|-----------------------------------|-----|
| 7.1 | Sequences of Real numbers | 246 |
| 7.1.1 | Definition of Sequence | 246 |
| 7.1.2 | Limit of a Sequence | 247 |
| 7.2 | Series and Convergence | 247 |
| 7.2.1 | Infinite Series | 247 |
| 7.2.2 | Convergent and Divergent Series | 248 |
| 7.2.3 | Geometric Series | 249 |
| 7.2.4 | n th -Term Test for Divergence | 250 |
| 7.3 | The Integral Test and p -Series | 251 |
| 7.3.1 | The Integral Test | 251 |
| 7.3.2 | p - Series and Harmonic Series | 252 |
| 7.4 | Comparison of Series | 252 |
| 7.4.1 | Direct Comparison Test | 252 |

CHAPTER 3

APPLICATIONS OF DIFFERENTIATION

Zaharah Wahid

The derivative of a function has many applications to problems in calculus. It may be used in curve sketching; solving maximum and minimum problems; solving distance; velocity, and acceleration problems; solving related rate problems; and approximating function values. After completing chapter 1 and 2 you should be able to solve many differential problems in engineering. Exercises included in this chapter cover a wide variety of daily events to show the relevance of calculus to students' majors.

At this point it is interesting to note how Islam has contributed to civilization. The two remarkable and renowned Muslim mathematicians by the name Muhammad Al-Karaji and Abu Abdullah Muhammad bin Musa Al-khwarizmi. They had introduced the theory of Algebraic Calculus. The adoption of the sign of zero had led to the so called "Arithmetic of positions". Also with the help of the Arab system of numbers, elementary methods of calculations were perfected.

In differential calculus we shall discuss the first and second derivatives of a function. However our focus here is on a function of one variable.

3.1 Linear Approximations and Newton's Method

3.1.1 Small Changes and Linear Approximations

The derivative of a function can be used to approximate certain function values with high degree of accuracy. The concept of the differential of the independent variable and the dependent variable will be discussed.

The definition of the derivative of a function $y = f(x)$ is

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x_1 + \Delta x) - f(x)}{\Delta x}$$

which represents the slope of the tangent line to the curve at a point $(x, f(x))$ is also called the slope of the curve. If Δx is very small ($\Delta x \neq 0$), then the slope of the tangent line is